On the History of Indian Mathematics, The procedure to Extract Square roots

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Āryabhaṭa's procedure, Bhāskara's explanation

Here is a translation of Āryabhaṭa's rule to extract square-roots:

> **Ab.2.4. One should divide, constantly, the non-square ⟨place⟩ by twice the square-root|**
> **When the square has been subtracted from the square ⟨place⟩, the quotient is the root in a different place‖**
>
> **bhāgaṃ hared avargān nityaṃ dviguṇena vargamūlena|**
> **vargād varge śuddhe labdhaṃ sthānāntare mūlam‖**

The procedure

In the following, we will consider that the digits forming a number are ordered from left to right: the first digit being the one standing in the highest place.

Step 1 Probably by trial and error, find the biggest square (a^2) smaller than the first digit, if the first digit stands in a place with a square power of ten. If this isn't the case, find the biggest square smaller than a two digit number.

Step 2 Subtract it from the last digit, and substitute the difference in place of the former digit. The square-root of this square (a) is the first digit of the square-root sought.

Step 3 Considering the next place to the right, divide the number formed by considering all the digits to the left of that place (that place included) by twice the partial square-root obtained. The quotient should be smaller than 10.

Step 4 Replace the dividend by the remainder of the division. The quotient is considered here to be the next digit of the square-root sought.

Step 5 Considering the next place to the right, subtract from the number formed by all the digits to the left of that place (that place included) the square of the quotient. Replace that number by the difference. Re-iterate the process starting from Step 3. The process ends when one cannot shift to the right anymore.

Table 1: An Example: Extracting the square-root of a three digit number

Āryabhaṭa's rule	Example: extracting the square root of 625	Extracting the square-root of $A = (a.10 + b)^2$
When subtracting the square from the square ⟨place⟩	The biggest square smaller than 6, which is the digit in the "highest square place", is 4. So that 2 is the first digit of the square-root to be extracted. This is how the number may have been set down: $v \quad av \quad v$ $6 \quad 2 \quad 5$ $-4 \quad - \quad -$ $2 \quad 2 \quad 5$	$A - a^2.10^2$ is computed. $a.10$ is the partial square-root extracted.
One should divide, constantly, the non-square ⟨place⟩ by twice the square-root.	22 is considered to be in the 'non-square' place. Twice the partial square root is $2 \times 2 = 4$. One performs the following division: $$\frac{22}{4} = 5 + \frac{2}{4}.$$ 5 is the quotient, it is the second digit of the square-root to be extracted. The partial square-root is, at this point: 25. The remainder of the division of 22 by 5 is set down in the place of the previously written digits: $av \quad v$ $2 \quad 5$	b is computed as the quotient of the division of the two higher digits by a^2. Then $A - a^2.10^2 - 2ab10$ is set down. $a.10 + b$ is the partial square-root extracted.

Āryabhaṭa's rule	Example: extracting the square root of 625	Extracting the square-root of $A = (a.10 + b)^2$
The quotient is the root in the next place. When subtracting the square from the square	The quotient is 5. The next place being a square-place, one subtracts the square of 5. $\begin{array}{cc} av & v \\ 2 & 5 \\ & -5^2 \\ & 0 \end{array}$	$A - a^2.10^2 - 2ab10 - b^2$ is computed.
The square root found is	25	$a.10 + b$

Problem

Extract with this method the square-root of 100100025 and of 88209 .