Text 1: Definition of a Square

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

He says the first half of an *āryā* to expose operations on squares (*varga-parikarman*):

Ab.2.3-ab A square is an equi-quadrilateral, and the area/result (*phala*) is the product of a couple of identicals|[1]

(...)

A specific equi-quadrilateral field is the named (*saṃjñin*), a square is the name (*saṃjñā*). For otherwise, in this case, (...) there is a possibility for the name 'square' to be ⟨given to⟩ all those ⟨fields⟩, even when ⟨they are⟩ undesirable.

⟨Question⟩

When is there,(...), a possibility for the name 'square' to be ⟨given to⟩ an undesirable specific equi-quadrilateral field?

It is replied: This kind of equi-quadrilateral with unequal diagonals (*asamakarṇa*) has ⟨that name⟩ (Figure 1), and this ⟨field made of⟩ two equi-trilateral fields (*dvisamatryaśrakṣetra*) placed as if upraised, has ⟨that name⟩ (Figure 2).

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[1] **One can understand the verse as meaning:**

> **A square is an equi-quadrilateral and the result which is the product of a couple of identical ⟨quantities⟩ |**

or

> **A square is an equi-quadrilateral and ⟨its⟩ area is the product of a couple of identical ⟨sides⟩ |**

It is probably ambiguous in order to collect all these signification. Previous translators of this verse have noted this ambiguity. See [Sengupta 1927; p.13], [Clark 1930; p.21], [Shukla 1976; p.34]. Bhāskara expounds the verse in both directions.

Figure 1

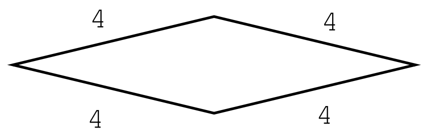
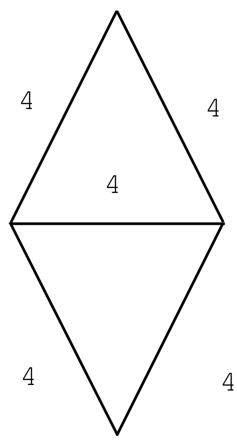


Figure 2



⟨Question⟩

What is wrong with the possibility for the name 'square' ⟨to be given to these fields⟩?

It is stated: 'And ⟨its⟩ area (*phala*) is the product of a couple of identical ⟨sides⟩', therefore, the product of a couple of identical ⟨sides⟩ should give the area, and such is not as wished in the above cases.

⟨Question⟩

When then ⟨is it right for the name 'square' to be given to a specific equi-quadrilateral⟩?

One should mention the diagonals (*karṇa*). A square is therefore a specific equi-diagonal-equi-quadrilateral field. Or perhaps, one intends to know the name 'square' for a kind of equi-quadrilateral field secondarily characterized (*upalakṣita*) indeed by two diagonals which have the same values (*saṅkhyā*).

Texte 2 : Fractions

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

The square of fractions (*bhinna*) is also just like this. However, when one has made separately the squares of the numerator (*aṃśa*) and denominator (*cheda*) quantities, that were made into the same kind (*sadṛṣa*), the result of the division of the square of the numerator quantity by the denominator quantity is the square of the fraction.

Example:

> **2. Tell me the square of six and one fourth and of one increased by one fifth |**
> **and of two minus one ninth ||**

Setting down: $\begin{array}{ccc} 6 & 1 & 2 \\ 1 & 1 & 1 \\ 4 & 5 & 9^{\circ} \end{array}$

Procedure (*karaṇa*): '⟨the whole number⟩ having the denominator for multiplier and increased by the numerator', therefore $\begin{array}{c} 25 \\ 4 \end{array}$ ⟨is obtained⟩.

Separately the square quantities of these denominator and numerator quantities are 16, 625. When one has divided the square of the numerator quantity by the square of the denominator quantity, the result (*labdha*) is $\begin{array}{c} 39 \\ 1 \\ 16 \end{array}$.

Likewise, ⟨the squares⟩ of the remaining ones also are, in due order, $\begin{array}{cc} 1 & 3 \\ 11 & 46 \\ 25 & 81 \end{array}$.

(...) The cube of a fraction is also just like that. An example:

> **4. Say, clearly, the cube-number of six, five, ten and eight who are computed with a decreasing by their**

respective parts|
If ⟨you have⟩ a clear knowledge in cube-computations||

Setting down:

5	4	9	7
5	4	9	7
6	5	10	8

The cubes obtained, according to ⟨the given⟩ numbers are:

198	110	970	488
107	74	299	191
216	125	1000	512

Text 3 : Area of a triangle

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

Now, in order to compute (*ānayana*) the area of a trilateral field (*tribhujakṣetra*), he says:

> Ab.2.6.ab **The bulk of the area of a trilateral is the product of half the base and the perpendicular|**

(...)

In this case there are three ⟨kinds of⟩ fields: equi⟨laterals⟩ (*sama*), isoceles (*dvisama*) and uneven ⟨trilaterals⟩ (*viṣama*).

(...)

An example:

> **1. Friend, ⟨tell⟩ the areas of equi⟨laterals⟩ whose sides (*bhujā*) are ⟨respectively⟩ seven, eight, and nine|**
> **And of an isoceles whose base (*bhū*) is six, and ears (*śravaṇa*) five||**

Setting down:

For the isoceles also, the setting down is:

Procedure:

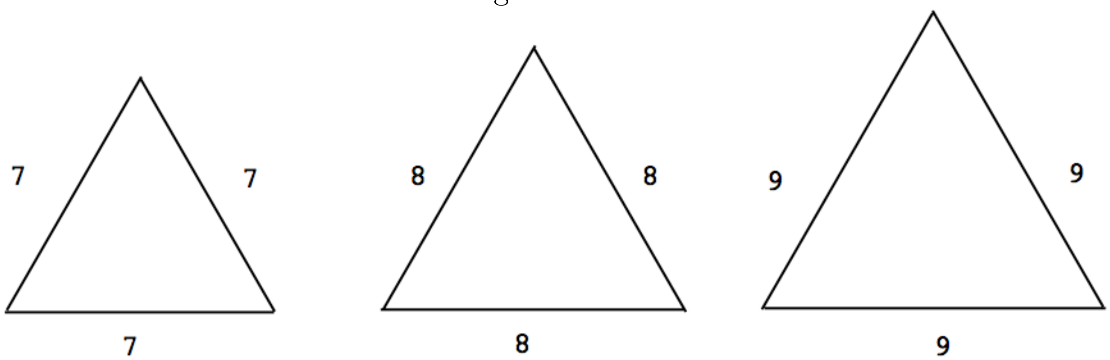
> "In an equi-trilateral field the location of the perpendicular is precisely equal[2]."

The section of the base (*ābādhā* [3]) which is half of the base is $\begin{array}{c}3\\1\\2\end{array}$.

[2] The use of *sama* (same) in this quotation is an elliptical way of expressing that the height sections the base in two *equal* segments.

[3] This is a technical term naming the two segments of the base delimited by the perpendicular.

Figure 4



These are three equi⟨laterals⟩.

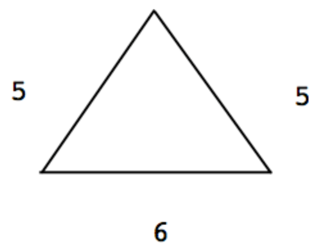


Figure 5

"That which precisely is the square of the base (*bhujā*) and the square of the upright side (*koṭī*) is the square of the hypotenuse (*karṇa*)".

That is, the square of the hypotenuse is ⟨the sum of⟩ the squares of both the base and the height. Therefore, when the square of the base is subtracted from the square of the hypotenuse, the remainder is the square of the perpendicular, that is $\begin{array}{c}36\\3\\4\end{array}$.

The perpendicular is $\begin{array}{c}36\\3\\4\end{array}$ *karaṇīs.*

Half the base also is $\begin{array}{c}12\\1\\4\end{array}$ *karaṇīs.* Therefore, since there is a product for two *karaṇīs*, the area of the field is obtained as "the product of half the side and the perpendicular", that is $\begin{array}{c}450\\3\\16\end{array}$ *karaṇīs*.

In due order, exactly in the same way, the area of the two remaining equi⟨laterals⟩ are ⟨respectively⟩ [768 *karaṇīs*], and $\begin{array}{c}1230\\3\\16\end{array}$ *karaṇīs*.

Since, for an isoceles trilateral also "The location of the perpendicular is precisely equal", a section of the base is 3. Using just the previous procedure, the perpendicular is 4. Using exactly the same procedure, the area is 12.

Text 4: Circles

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

Now, in order to compute (*ānayana*) the area of a circular field (*vṛttakṣetraphala*, he states:

> **Ab.2.7.ab. Half of the even circumference multiplied by the semi-diameter, only, is the area of a circle|**

(...)

⟨Objection⟩
This is not so, because an other method is heard of elsewhere: 'The square of the semi-diameter with three as multiplier is the computation.'

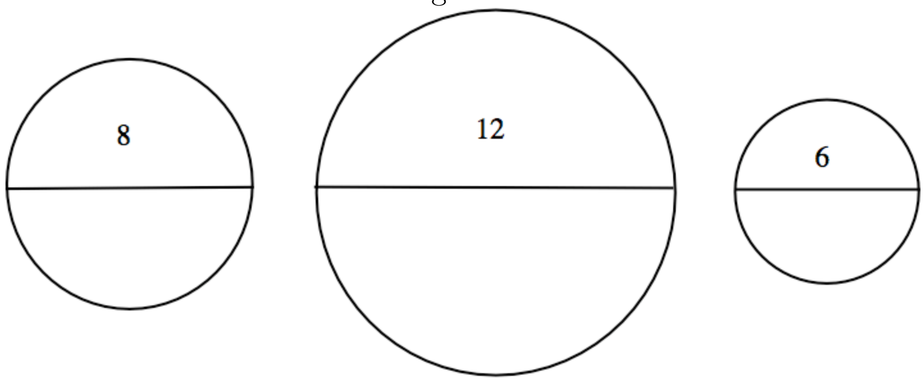
This particular method is not accurate (*sūkṣma*) , but practical (*vyāvahārika*). Therefore, there is only one method. There is no other for a computation in accurate mathematics (*sūkṣmagaṇita*).

An example:

> **1. I see accurately diameters (*viṣkambha*) ⟨whose lengths⟩**
> **are eight, twelve and six. |**
> **Tell me, separately, the circumference (*paridhi*) aand**
> **the area (*phala*) of those evenly circular ⟨fields⟩ (*samavṛtta*)||**

Setting down-8, 12, 6

Figure 12



The circumferences obtained for these ⟨diameters⟩ by means of a Rule of Three, ⟨which uses⟩ as measure and fruit ⟨quantities⟩ (*pramāṇaphala*) the diameter and the circumference to be told [in Ab.2.10], are[4], in due order, $\begin{array}{ccc} 25 & 37 & 18 \\ 83 & 437 & 531 \\ 625 & 625 & 625 \end{array}$.

Procedure when computing the area: ‘half of the even circumference’, the semi-diameter produced is 4.

Half of the even circumference of that ⟨circle⟩, which is $\begin{array}{c} 12 \\ 354 \\ 625 \end{array}$, is multiplied by this very ⟨semi-diameter⟩.

[4]Knowing that a circle of diameter 20 000 has a circumference of 62832, we then have:

$$\mathcal{C}_1 = \frac{8 \times 62832}{20000} = \frac{15708}{625}$$

$$\mathcal{C}_2 = \frac{12 \times 62832}{20000} = \frac{23562}{625}$$

$$\mathcal{C}_\ni = \frac{6 \times 62832}{20000} = \frac{11781}{625}$$

.

The area of the circle produced is $\begin{matrix} 50 \\ 166 \\ 625 \end{matrix}$.

With just that procedure, the areas of the two remaining circumferences, are, in due order, $\begin{matrix} 113 & 28 \\ 61 & 343 \\ 625 & 1250 \end{matrix}$.

In order to compute (*ānayana*) an evenly-circular ⟨field⟩ (*samavṛtta*) with a Rule of Three, he states:

> **10. A hundred increased by four, multiplied by eight, and also sixty-two thousand|**
> **Is an approximate circumference of a circle whose diameter is two *ayutas*[5]||**

(...)

'Approximate' (*āsanna*) is near.

⟨Question⟩
What is it an approximation (*āsanna*) of?

Of the exact (*sūkṣma*) circumference.

⟨Question⟩
How is it known as an approximation of an exact ⟨value⟩ (*sūkṣmasya āsanna*) and not indeed as an approximation of a practical ⟨value⟩ (*vyāvahārikasya āsanna*), as long as the determination of what has been heard ⟨in the verse⟩ is the same (*tulya*) ⟨whether the value approximated is⟩ exact or practical (i.e. in all cases the value is an approximation).

There is no mistake. This is just a doubt (*sandeha*). The ⟨following⟩ knowledge stands for all doubts: 'A specific meaning arises from interpretation, [by no means does ⟨a rule⟩ become invalid (*a-lakṣaṇa*) because of a doubt]'.

Therefore we are giving the interpretation that it is the approximation of an exact ⟨value⟩.

Or else, (...) if it were an approximation of a practical ⟨value⟩, then the circumference ⟨obtained⟩ from that practical ⟨value⟩ would be even worse. No one would make an effort ⟨leading to something⟩ worse. Therefore it has been surely established (*nyāyasiddham*) that it is an approximation of an exact ⟨value⟩.

⟨Question⟩
Now, why is the approximate circumference told, and not indeed the correct (*sphuṭaparidhi*) circumference itself?

[5] **An *ayuta* is the name of ten thousand.**

They[6] believe the following: There is no such method (*upāya*) by which the exact circumference is computed.

⟨Objection⟩

But here it is:

> **The *karaṇī* of ten times the square of the diameter produces the circumference of the circle |**

In this case also, it is merely a tradition (*āgama*) and not a proof (*upapatti*): 'the circumference of a unity-diameter ⟨circle⟩ is ten *karaṇīs*.'

⟨Objection⟩

Now, some think that the circumference of a field with a unity-diameter when measured just by the eye (*pratyakṣa*) is ten *karaṇīs*.

This is not so because *karaṇīs* do not have a statable size.

⟨Objection⟩

The circumference ⟨of the field⟩ with that ⟨unity-⟩diameter, when enclosed by the diagonal, whose *karaṇīs* are precisely ten, of a rectangular field whose width (*vistāra*) and length (*āyāta*) are respectively one and three, that ⟨circumference⟩ has that size (i.e. it measures ten *karaṇīs*).

But that also should be established (*sādhya*).

[6]This anonymous collective voice is used from time to time in this commentary, and must be referring to scholars who had commented on this point.

Text 5 : Chords in a Circle

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

An example-

> **How much are the sizes of the half-chords on a semi-diameter measuring *Vasu* (8)-Fire (*dahana* 3)-*Kṛta* (4)-Fire (*hutāśana* 3)?|**

⟨Half-chords of⟩ the unit arc (*kāṣṭha*) which is half a *rāśi* are produced. The semi-diameter is 3438.

Procedure- Having drawn a circle (*maṇḍala*) with a pair of compasses (*kakarṭa*) whose ⟨opening⟩ is equal to the semi-diameter determined by a size as large as ⟨desired⟩, one should divide that ⟨circle⟩ into twelve. And these twelfth parts should be regarded as "*rāśis*". Now, in the circle which is divided into twelve, in the east one should make a line which has the form of a chord, and which penetrates (*avagāhinī*) ⟨the circle at⟩ the tips of two *rāśis* from south to north. Likewise also in the western part. And in exactly the same way in the southern and northern parts also, one should make chords extending from east to west. And furthermore in the eastern, western, southern and northern directions, in exactly the same way, one should make lines which penetrate ⟨the circle at⟩ the tips of four *rāśis*. Then they should be made into trilateral ⟨by drawing the diagonals of the rectangles obtained⟩.

And then a field produced by a circumference is drawn with a pair of compasses with a stick (*vartikā*) fastened to the opening (*mukha*). In the field drawn in this way all is to be shown.

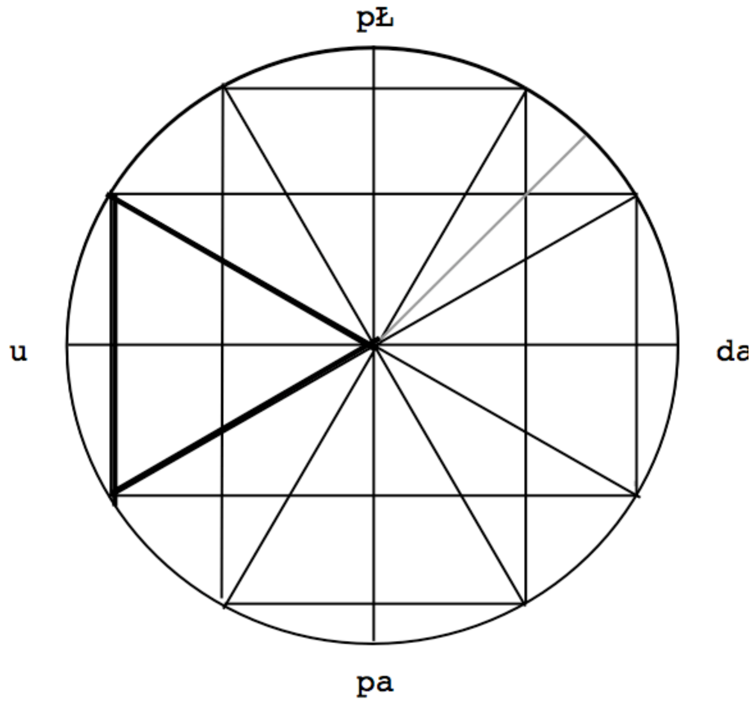


Figure 29

In this drawing (*ālekhya* i.e, when the unit arc is half a *rāśi*) the [whole] chord of four unit arcs (*kāṣṭha*) is equal to the semi-diameter. Half of that is the ⟨half-⟩chord (*jyā*) of two unit arcs. And that is 1719.

This is the base (*bhujā*), the semi-diameter is the hypotenuse (*karṇa*), therefore the perpendicular (*avalambaka*) is the root of the difference of the squares of the hypotenuse and the base. That exactly is the ⟨half-⟩chord of four unit arcs. And that is 2978. When one has subtracted this from the semi-diameter, the remainder is the arrow of ⟨the half-chord of⟩ two unit arcs. The hypotenuse is the root of the sum of the squares of the ⟨half-⟩chord of two unit arcs and the arrow. And that precisely is the [whole] chord of two unit arcs, which is 1780. Half of that is the ⟨half-⟩chord of one unit arc, 890.

Text 6: Elementary geometrical constructions

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

> **Ab.2.13. A circle should be brought about with a pair of compasses, and a trilateral and a quadrilateral each ⟨are brought about⟩ with two diagonals|**
> **Flat ground should be brought about with water, verticality (litterally: top and bottom) with just a plumb-line||**

(...)

A circular field is brought about with a *bhrama*. With the word *bhrama* a pair of compasses (*karkaṭa*) is understood. With that pair of compasses an evenly circular field is delimited by the size of the out-line (*parilekhā*).

⟨As for:⟩ "**Both a trilateral and a quadrilateral with diagonals**". A trilateral field and a quadrilateral field should each be brought about with two diagonals. First a trilateral:

Having stretched a string (*sūtra*) on level ground one should make a line (*rekha*). And that is:

Figure 30

Here, with a pair of compasses (*karkaṭaka*) which is placed on both tips ⟨of the line⟩, a fish should be produced.

A perpendicular is a second string which goes from the mouth to the tail of this ⟨fish⟩:

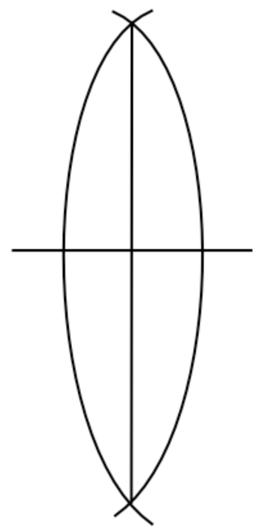


Figure 31

Having appointed one tip of a string on the extremity ⟨of the fish⟩, having appointed the second tip ⟨of the string⟩ firmly on the tip of the base, one should make a line. On the second tip ⟨of the base⟩, too, it is just in that way. In this way, there are two diagonal strings. With those two diagonal strings a trilateral is brought about:

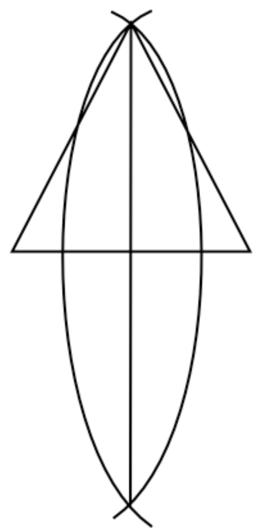


Figure 32

In ⟨the case of⟩ a quadrilateral, one should stretch obliquely a string which is equal to [the diagonal of] the desired quadrilateral. And that string is:

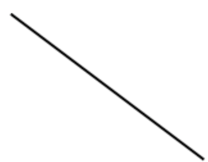


Figure 33

One should stretch obliquely the second ⟨string⟩ too, a cross (*svastika*) is produced from the middle of that ⟨first string⟩. And therefore there are two diagonal strings:

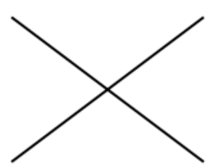
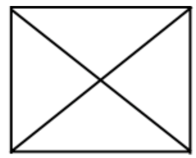


Figure 34

The sides (*pārśva*) of these two ⟨strings⟩ are filled in, ⟨and⟩ a quadrilateral field is produced:

Figure 35 

Text 7 : Decimal place value notation

A portion of Bhāskara's commentary (628 A. D.) on the *Āryabhaṭīya* (499. A. D.)

In order to assign places (*sthāna*) to numbers (*saṅkhyā*), he states:

Ab.2.2. One and ten and a hundred
And one thousand, now ten thousand and a hundred thousand, in the same way a million|
Ten million, a hundred million, and a thousand million.
A place should be ten times the ⟨previous⟩ place||

(...)

Here this may be asked: What is the power (*śakti*) of the places, ⟨that power with⟩ which one unit becomes ten, a hundred, and a thousand? And truly if this power of places existed, purchasers would have shares in especially desired commodities. And according to ⟨their⟩ wish what is purchased would be abundant or scarce.[7] And if this was so, there would be the unexpected possibility for things to be different in worldly affairs (*lokavyavahāra*).

(...)

And the setting down of places is:

o o o o o o o o o oo

[7] In other words, if the place decided the value, in the world as well as in the treatise, one could buy a small amount, and then increase it afterwards, by simply changing its place.